

2.1 Derivative Basics: Form C Key

Tutor copy. Every answer was checked by symbolic computation (SymPy, `verify_set.py`). All problems are original; see the notes file for provenance. Prepared with assistance from Claude, an AI model made by Anthropic.

General grading rules

- **Answer-only items** (Q1, Q9, Q13): all or nothing, except where a row below says otherwise.
- **Value vs. derivative.** Giving a derivative where a function value was asked (or the reverse) earns 0 for that part.
- **Show-work items** (Parts II–IV other than Q9): a bare correct answer earns at most half credit.
- **Definition items** (Q6, Q7): the problem states the required method. A correct answer by differentiation rules earns at most 2 of the 8 points.
- **Method is not graded for efficiency.** A correct answer reached by long but valid work gets full credit; the time it costs is the penalty.

Form C

Item	Answer	Rubric / trap
1(a)	-1	<i>Trap:</i> the tangent drawn there invites the slope, -2; earns 0.
1(b)	2	<i>Trap:</i> giving the height $f(4) = -1$; earns 0.
1(c)	0	<i>Trap:</i> giving the height -2 of the low point; earns 0.
1(d)	DNE	Corner at $(1, 2)$: one-sided slopes -1 and -4. <i>Trap:</i> a slope of -1 or -4, or deciding the derivative exists because $(1, 2)$ is not a high or low point.
1(e)	$(-5, 1)$ and $(1, 3)$	3 pts, all or nothing; union notation is fine. Endpoints may not be included. <i>Trap:</i> $(-5, 3)$, which includes $x = 1$, where f' does not exist.
1(f)	$f'(-1) < 0 < f'(4)$	$f'(-1) = -1$, $f'(4) = 2$. 3 pts, all or nothing.
2	$y = 1$	1 pt $f(2) = 1$; 2 pts $f'(x) = 3x^2 - 6x$; 1 pt $f'(2) = 0$; 1 pt equation. $y - 1 = 0(x - 2)$ is accepted. <i>Trap:</i> a slope of 0 makes students think there is no tangent line, or they write the vertical line $x = 2$ (0 for the equation).
3	$y = 3x$	1 pt $r(0) = 0$; 2 pts $r'(x)$; 1 pt $r'(0) = 3$; 1 pt equation in any correct form. <i>Trap:</i> $r'(x) = 3 \cos x + 2x$, so $r'(0) = 3 \cos 0 = 3$. Using $\sin 0$ for $\cos 0$ gives slope 0; earns 2 of 5.
4(a)	3	3 pts. Simplifies to $y = 3x - 13$. <i>Trap:</i> -2, the coefficient in the familiar slope position; earns 0.
4(b)	-4	2 pts. Substitute $x = 3$. <i>Trap:</i> 4 (sign error moving the +4); earns 0.
5	4	<i>Trap:</i> h' is printed and invites $h'(3) = -9$; earns 0.
6	$f'(x) = \frac{2}{(3-x)^2}$	2 pts $f(x+h) = \frac{2}{3-(x+h)} = \frac{2}{3-x-h}$, substituted correctly and in parentheses; 2 pts common denominator, numerator $2(3-x) - 2(3-x-h) = 2h$; 2 pts cancel h to get $\frac{2}{(3-x-h)(3-x)}$; 2 pts let $h \rightarrow 0$. <i>Trap:</i> writing $f(x) + h$ for $f(x+h)$, or losing a sign across the subtraction. Rules only: at most 2.

Item	Answer	Rubric / trap
7	$\frac{1}{6}$	1 pt $g(9) = 3$; 1 pt setup $\frac{\sqrt{x}-3}{x-9}$; 3 pts simplify to $\frac{1}{\sqrt{x}+3}$. Multiplying top and bottom by $\sqrt{x}+3$ works, and so does factoring $x-9 = (\sqrt{x}-3)(\sqrt{x}+3)$; either gives $\frac{1}{\sqrt{x}+3}$. No particular method is required. 3 pts value $\frac{1}{6}$. <i>Trap</i> : stopping at $\frac{0}{0}$, or rationalizing and then substituting before canceling. Rules only: at most 2.
8(a)	4	$\frac{1}{3}f'(2)$ for $f(x) = x^3$: $\frac{1}{3} \cdot 12 = 4$. <i>Trap</i> : 12, ignoring the $3h$; earns 0. 3 pts for recognizing which derivative the limit is (or a correct algebraic setup), 3 pts for the value.
8(b)	0	$\sin \frac{\pi}{2} = 1$, so this is the derivative of $\sin$ at $\frac{\pi}{2}$: $\cos \frac{\pi}{2} = 0$. <i>Trap</i> : “0/0, DNE” or 1; earns 0. 3 pts for recognizing which derivative the limit is (or a correct algebraic setup), 3 pts for the value.
9	(a) 0 (b) 0	3 each. <i>Traps</i> : (a) $3\pi^2$; (b) $f'(\pi) = -1 + 2\pi$. In (b), $m(x)$ is the same number for every x , so $m'(x) = 0$; the problem says “for every x ”.
10(a)	$12x^3 + 5x^{-2}$	1 pt rewriting $-5x^{-1}$; 2 pts derivative. <i>Trap</i> : $-5 \cdot (-1) = +5$.
10(b)	$6x$	1 pt rewriting $3x^2 + 1$ (for $x \neq 0$); 2 pts derivative. <i>Trap</i> : dividing only the first term by $2x$ gives $3x^2 + 2x$; differentiating top and bottom separately earns 0.
10(c)	$2x^{-3/2} - \sin x$	1 pt rewriting $-4x^{-1/2}$; 2 pts derivative. <i>Trap</i> : $-4 \cdot (-\frac{1}{2}) = +2$; the exponent $-\frac{3}{2}$.
10(d)	$18x - 5$	1 pt rewriting $9x^2 - 5x + 1$; 2 pts derivative. <i>Trap</i> : $(3x - 1)^2 \neq 9x^2 + 1$.
11	2	3 pts $f'(x) = 2 \cos x + 3$; 2 pts value. <i>Trap</i> : $\cos \frac{2\pi}{3} = -\frac{1}{2}$. Using $+\frac{1}{2}$ gives 4; earns 3.
12	$x = \frac{2\pi}{3}$	2 pts $f'(x) = \sqrt{3} - 2 \sin x$; 2 pts $\sin x = \frac{\sqrt{3}}{2}$; 2 pts the solutions in the interval. <i>Trap</i> : $\frac{\pi}{3}$ also solves $\sin x = \frac{\sqrt{3}}{2}$ but lies outside $(\frac{\pi}{2}, 2\pi)$; listing it costs 2.
13(a)	I	u' is a line crossing zero at $x = \frac{5}{2}$, where u turns, with the sign of u 's slope on each side. <i>Trap</i> : IV, the same line with the opposite sign, or a parabola because u is a parabola.
13(b)	III	v' is a parabola with zeros at $x = -\frac{5}{2}$ and $x = \frac{3}{2}$, where v turns. <i>Trap</i> : II, which has the right zeros and the wrong signs.
14	see sketch description	$k' = -2$ for $x < -2$; $k' = 1$ for $-2 < x < 1$; $k' = 0$ for $x > 1$. Open circles at the ends of each segment, at $x = -2$ and $x = 1$. 2 pts per horizontal piece (correct height and interval); -1 once if the open circles are missing or replaced by closed dots. <i>Trap</i> : drawing k 's shape again, or joining the pieces with vertical segments.

Points: $14 + 5 + 5 + 5 + 2 + 8 + 8 + 12 + 6 + 12 + 5 + 6 + 6 + 6 = 100$.